High-Resolution CVS: Generalized Covariance Measure

SM6.5

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SUMMARY

Traditional velocity analysis techniques use time gates and attributes such as semblance to detect event velocity and zero offset time. The use of a recent attribute, covariance measure which is defined through the use of the eigenvalues of the covariance matrix, increases the resolution in time and velocity. Each element in the covariance matrix is the zero lag of the cross correlation between two gates belonging to a trace pair and therefore CM is related to energy normalized cross correlation sum. A covariance measure can be defined from energy normalized cross correlation sum. Since semblance and energy normalized cross correlation sum are related, a covariance measure can also be defined from semblance and is the most economical of the three methods.

INTRODUCTION

Semblance, unnormalized cross correlation sum, statistically normalized cross correlation sum, energy normalized cross correlation sum (ENCCS) and some other attributes such as mean amplitude have been used (Neidell and Taner, 1971) as attributes. Recently, new techniques which increase the resolution of the attributes have emerged (Sguazzero et al.,1987, Key et al,1987, Biondi and Kostov,1988, and Key and Smithson, 1990). Key and Smithson(1990) used covariance measure (CM) as the attribute. Their assessment that CM has better resolution than semblance has prompted this study.

REVIEW

Semblance

Attribute semblance is well known (Neidell and Taner, 1971). The signal to noise variance ratio (or, the signal to noise energy ratio) can be obtained from semblance s through $SNR_E = s/(1-s)$. The square root of SNR_E is the signal to noise RMS amplitude ratio (SNR_{rms}).

Energy normalized cross correlation sum

Energy normalized cross correlation sum (ENCCS or simply c) defined by Neidell and Taner (1971) can be interpreted as the ratio of the mean (C) of the off-diagonal elements in the "covariance matrix" defined by Key and Smithson(1990) to the mean (A) of the elements of the main diagonal:

$$c = \frac{C}{A} \quad . \tag{1}$$

Covariance Measure

Key and Smithson (1990) have introduced covariance measure, CM. The definition of CM is

$$CM_{eig} = (S/N)\ \rho^{N_x} \tag{2}$$

Here S/N is the signal to noise variance ratio defined from the eigenvalues (and is observed through tests to be equal to SNR_E defined from semblance, aside from a factor which is equal to N_x, the fold of the CDP), and rho is a factor related to the inequality of the eigenvalues. Rho is defined as the natural logarithm of the ratio of the arithmetical mean (a) to the geometrical mean (g) of the eigenvalues:

$$\rho = \ln\left(\frac{a}{g}\right) \tag{3}$$

Their basic assumption is that there is only one event per scan (one event per data matrix). Even if there are many events, at the velocity of one of the events, all others are incorrectly moved-out and appear as noise. Therefore for the correct velocity there is always one major eigenvalue. This fact is the backbone of CM and is the reason for the second factor in it. Key and Smithson use a partial stacking scheme and so reduce the fold of the CDP (the power of rho) by some factor, for example 6. Note that a large power (like 48) in CM would give serious numerical problems, if the partial stacking route is not chosen. Similarly, the geometrical mean of 48 numbers is likely to give numerical problems.

GENERALIZATION OF COVARIANCE MEASURE TO ENCCS AND TO SEMBLANCE

For the pure signal case the covariance matrix is a singular matrix and CM calculations may get into trouble. The stability of eigenvalue solution can be obtained by adding white noise to the main diagonal of the covariance matrix. Spurious peaks were obtained even after this for data with poor signal to noise ratio. The study of the covariance matrices for such records revealed two facts : a)-there are many negative elements in the covariance matrix due to either random noise or incorrect scan velocity , b)-spurious peaks occur when there are very small eigenvalues. Since, for trajectories aligning the signal there should not be any negative element in the covariance matrix, one might choose to zero the negative elements before eigenvalue calculations (due to one event assumption, this is not harmful). Also, one may set a lower bound for the eigenvalues. These two modifications resulted in significant improvements in the CM runs, which prompted the following question: what if we replaced every off-diagonal element in the covariance matrix with their mean(C) and every diagonal element with their mean(A). In this case the eigenvalues of the matrix are

$$\lambda_1 = A-C + N_x C, \quad \lambda_2=\lambda_3=\ldots=\lambda_{N_x} = A-C \ . \tag{4}$$

Note that noise variance σ_n^2 defined through covariance measure method (Smithson and Key, 1990) becomes equal to A-C and signal variance σ_s^2 becomes equal to C, and therefore S/N becomes

$$S/N = \frac{c}{1-c} \tag{5}$$

where c=A/C is the ENCCS. This equation has the same form as the equation used to define SNR_E from semblance. The arithmetical mean of the eigenvalues is equal to A and the geometrical mean is equal to

$$\left((A-C)^{N_x-1} \ (A-C+N_xC) \right)^{\frac{1}{N_x}} \quad . \qquad (6)$$

Therefore the ratio of the geometrical mean to the arithmetical mean is

$$\frac{1}{\left((1-c)^{N_x-1} \ (1+(N_x-1)c) \right)^{\frac{1}{N_x}}} \quad . \qquad (7)$$

This ratio goes to infinity for good signal (c= 1), and goes to one when there is no signal (c = 0), and is mostly needed for velocity resolution for poor data. Second factor in Eq.7 does not exhibit much sensitivity to c (due to root N_x) and can be dropped. Then, Eq. 7 could be approximated with $1/(1-c)$ for the range $0<c<1$. And rho becomes

$$\rho \approx \ln\left(\frac{1}{1-c}\right) \quad . \qquad (8)$$

Note that rho goes to c as c goes to zero, therefore taking its N_xth power during CM calculation brings high sensitivity to c for noisy data.
Since semblance and ENCCS are very close to each other, rho corresponding to semblance could be obtained from here by replacing c with semblance s:

$$\rho \approx \ln\left(\frac{1}{1-s}\right) \quad . \qquad (9)$$

Indeed there is no reason to insist on N_x as the power in CM. A small number like 8 might be enough to replace N_x. A simpler form of CM would be

$$CM = (S/N)\ \rho^8 \qquad (10)$$

It can be used for all three methods, eigenvalue technique, ENCCS and semblance.

Figure 1 is a single event with virtually no noise (SNR_E =1600 in 44 ms gate around the main peak, time 2 sec, velocity 2700 m/s far offset 1400 meters, 48 fold CDP, signal and noise band pass are 10, 60 Hz, time gate size in the velocity analysis is 44 ms). Figure 2 compares following attributes: semblance, ENCCS, SNR_E obtained from semblance, three different CM values obtained from eigenvalues, ENCCS and semblance. Partial stacking is used in the eigenvalue technique to reduce the dimension of the matrix from 48 to 8. In ENCCS and semblance runs 48 traces are used to calculate the semblance and ENCCS, yet number 8 is used as power of rho instead of 48.
Note the immediate gain in velocity resolution by going from semblance to SNR_E. Note also the large temporal extend of the event in semblance and ENCCS which is due to 44 ms gate length. Slightly larger temporal extend of CM_{eig} compared to other CMs is due to 0.1 percent white noise added to the covariance matrix, before eigenvalues are calculated, and can be made smaller by using a smaller value for the white noise.

Figure 3 is the same single event with some band limited noise (SNR_E=1 in 44 ms gate). Figure 4 compares the various attributes ran on Figure 3. Note the squareness of the semblance and ENCCS contours. This is due to noise and 44 ms time gate. Also note that SNR and semblance are now almost the same resolution. This is due to noise. However, using any of the CM measures increases resolution because of rho factor (power used is 8).

CONCLUSIONS

For high SNR, high velocity resolution in CM is mainly due to SNR term but not due to the use of eigenvalues per se. For low SNR, the resolution comes from the factor containing rho. Due to CM's relationship to crosscorrelations, a theoretical CM can be obtained from ENCCS without solving for the eigenvalues. Since ENCCS and semblance measure very much the same quantity, CM can be calculated from the semblance as well by replacing ENCCS with semblance, allowing us to generalize the covariance measure function to semblance as well as ENCCS. Obviously, semblance is the most economical. Both ENCCS and semblance calculate signal and noise energy at each scan. As long as we assume that there is one event at each trajectory and devise our formula accordingly there is no extra benefit derived from eigenvalue technique.

ACKNOWLEDGMENTS

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Figure 1, A single event with a very small amount of noise; t_0=2.000 s, v=2700 m/s, SNR_E=1600.

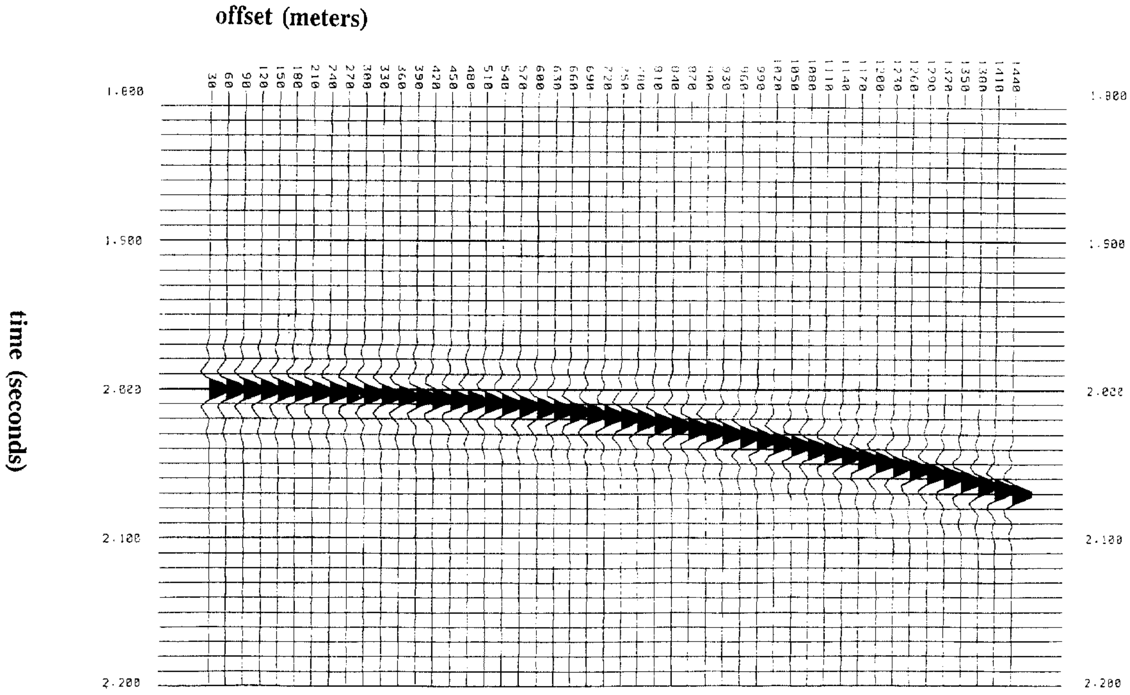

Figure 2, Comparison of various attributes for a single event in Figure 1. Temporal gate length=0.044 s.

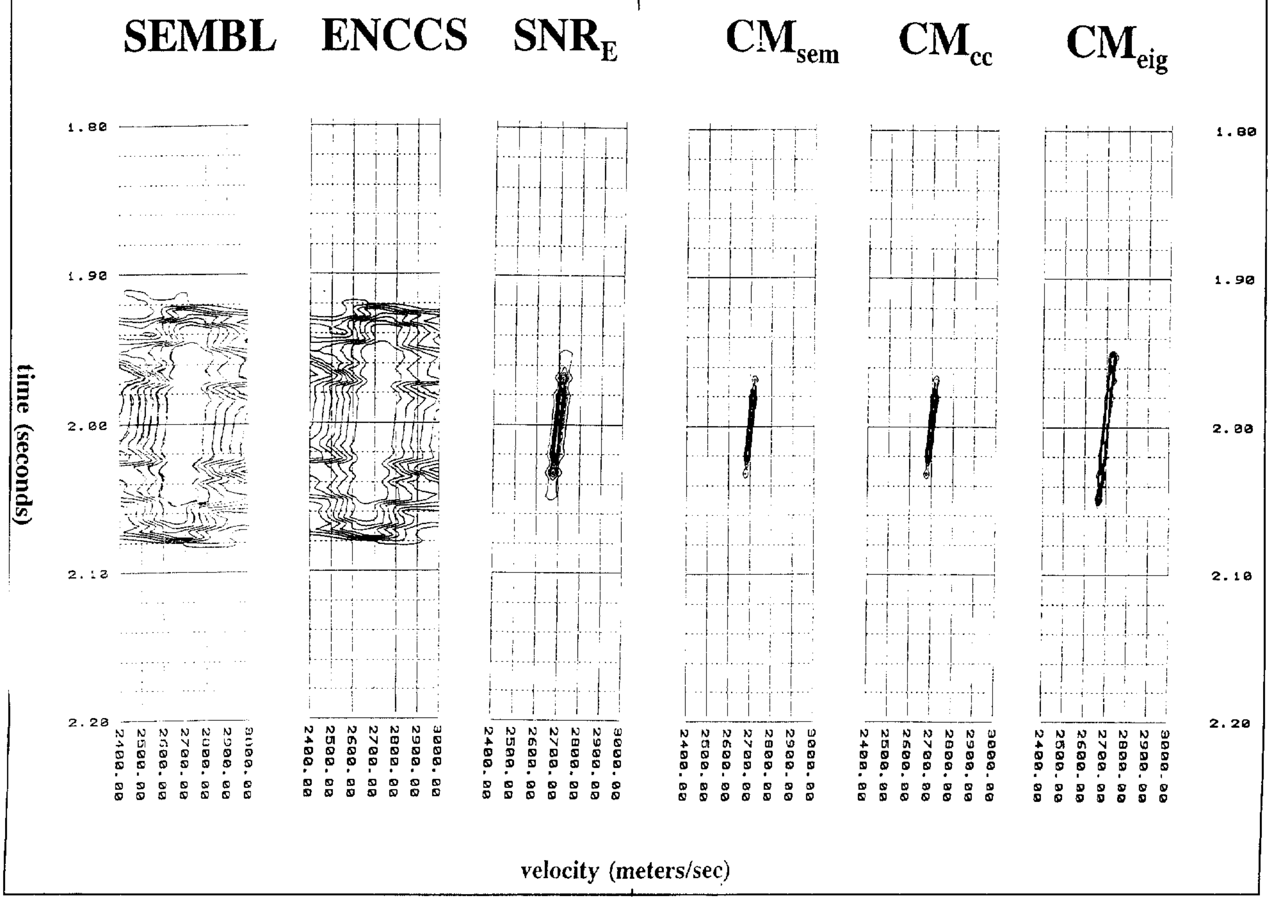

Figure 3, A single event with moderate amounts of noise; t_0=2.000 s, v=2700 m/s , SNR_E=1.

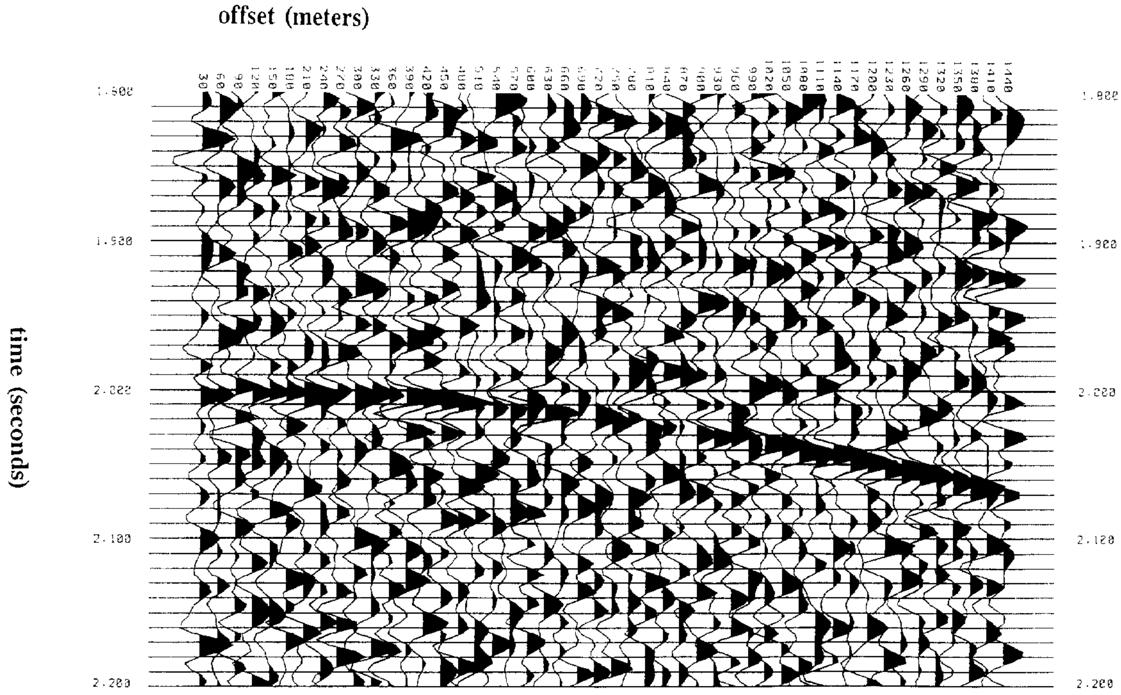

Figure 4, Comparison of various attributes for a single event in Figure 3. Temporal gate length is 44 s.